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14. ABSTRACT The focus of this project is about numerical solutions of optimal control problem using the analysis of variance (ANOVA) analysis. The impact of random parameter dependent boundary conditions on the solutions of a class of nonlinear partial differential equations (PDEs) is considered. Because the boundary conditions are random field, the PDE becomes stochastic PDE. The concepts of effective dimensions are used to determine the accuracy of the ANOVA expansions. Demonstrations are given to show that whenever truncated ANOVA expansions of functionals provide accurate approximations, optimizers found through a simple surrogate optimization strategy are also relatively accurate.

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NUMERICAL SOLUTIONS FOR OPTIMAL CONTROL PROBLEMS UNDER SPDE CONSTRAINTS

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Abstract

The primary source of aircraft noise is the fan noise from the engines; natural approaches to reducing this noise involve acoustic shape optimization of the inlet and impedance optimization of the liner. This project will use optimal control to systematically determine the inlet shape and the linear material impedance factor that minimize the fan noise.

A novel feature of this approach is that we automatically incorporate uncertainty and data measurement errors. Specifically we assume that the acoustic wave number is a random variable/field instead of a constant. This means that the computed answers are valid, not merely for a single configuration, but for a wide range. Our numerical results show significant noise reduction with the optimal impedance factor.

Since the wave number is random, the underlying partial differential equation–Helmholtz equation in our case, is a stochastic partial differential equation. In this project, we have constructed efficient Monte Carlo methods as well as stochastic finite element methods to solve stochastic partial differential equations. Rigorous error estimates are obtained and numerical simulations are conducted to support the error analysis.

Formulation of the optimal control problem

In this research, we treat liner impedance optimization as an optimal control and parameter estimation problem. The parameter is the acoustic impedance factor of the acoustic liner. We

define a cost function that reflects the amount of noise radiated from the engine inlet. The parameter estimation problem then is to seek the parameter that minimizes the cost function. The geometry of the domain in which the control problem is posed has the generic shape represented in Figure 1. The modal composition of the noise source is supposed to be known on the source plane Γ_1. The nacelle boundary is made up of two parts, the first part being the interior boundary Γ_2 to which some acoustic liner material is attached, and the second part being Γ_3 that constitutes the rest of boundary of the nacelle geometry. The boundary Γ_4 is assumed to be sufficiently far from the noise source so that the Sommerfeld radiation boundary condition holds. The nacelle symmetry axis is denoted by Γ_5. We assume that the mean flow is zero. Then the acoustic pressure u satisfies the Helmholtz equation
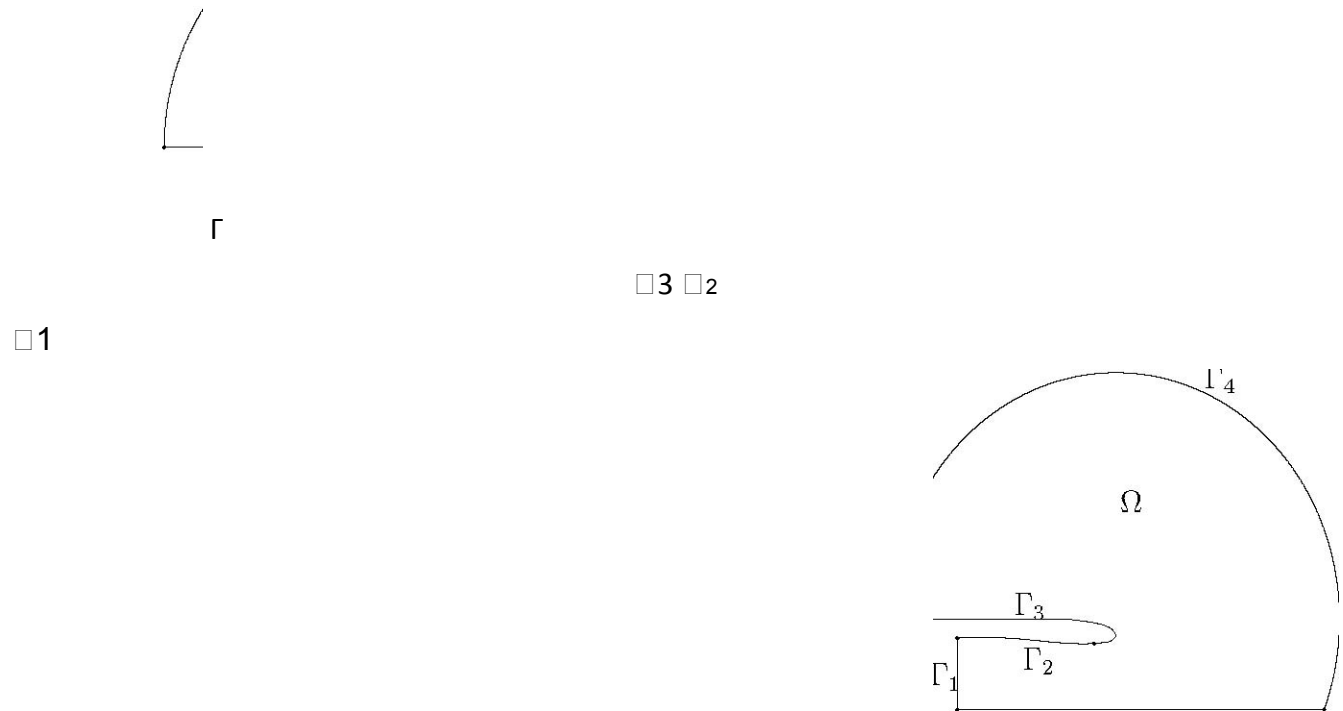
(1) $u + k^2 u = 0$ on Γ

5 Figure 1. The computational domain subject to the following

boundary conditions on the boundary $\partial\Gamma$ of Γ:

$$u \mid_1 = g,$$

$$\rho u \, \rho n + \frac{ik}{u} \mid_2 = 0,$$

(2) $\rho u \, \rho n \mid_3$

$$iku = 0, 0,$$

$$\rho u \, \rho n + \mid_4 =$$

ρu

$\left. \dfrac{\partial \phi}{\partial n} \right|_s = 0.$

Here k is the wavenumber and   the impedance factor, which is complex, and whose real part is resistance and the imaginary part is reactance. Both the dependent and the independent variables in the above equations are supposed to be properly non-dimensionalized.

We first assume that k is a constant. Then optimization problem consists in finding the parameter   so that the least amount of noise propagates to the far field, while respecting some constraints on the boundary shape. More specifically, we want to find   so that the cost functional

$$(3) \quad J(\ , u) = \eta \int u^2 \, d\Gamma + \xi \int |\leq u|^2 \, d\Gamma + \tau |\ - \ _0|^2$$

is minimized, where η, ξ and τ are penalty parameters, $\eta \ \geq 0$, $\xi \geq 0$, $\tau \geq 0$, and $_0$ is a given complex number.

Because of the varying operating conditions and errors in measurement, k is not likely a constant, instead it may be a random field. For simplicity we assume that it is a random variable with small variance:

$$k = k0 + \chi \quad \text{where } k0 \text{ is a constant and } \quad \text{is a random}$$

variable with zero expectation, i.e., $E\ = 0$. In this case we construct the new cost function as

$$\hat{J}(\ ) = E(J(\ , \ )).$$

The optimal control problem becomes finding λ such that

$$\hat{J}(\lambda\ ) = \min \hat{J}(\lambda).$$

Clearly such an approach makes the control models more complex, but much more flexible, realistic and practical. Because the effects of data uncertainty are built into the model, we expect to see that the optimal controls will be much less sensitive to changes in the model parameters. The result is much greater robustness for the optimal controls.

Fast Monte Carlo simulation for the optimal control problem
The main challenge of solving the optimal control problem under uncertainty is the need of solving large number the state equations when using Monte Carlo method to evaluate the cost function. We have developed a fast numerical algorithm to obtain the numerical solution for the optimal control problem under uncertainty ([1, 2]) . The main idea is to reduce the variance by sampling the residual of the Taylor expansion for cost function in terms of the random wave number.

We have performed a variety of numerical experiments to show that the choices of optimal parameters result in reducing the noise level significantly. Here we only present one example to illustrate the numerical procedure. We choose the wave number $k0 = 2\alpha$, $\chi = 0.2$ and   a random variable with uniform distribution $U (-1, 1)$. The source function is defined by $g(y) = 20 + \exp(3y)\sin(10\alpha y)$. To evaluate the cost function, we combine the sensitive derivative Monte Carlo (SDMC) developed in [1] with the stratified Monte Carlo method to accelerate the evaluation of the cost function (see [2] for detailed implementation). Figure 1 is the contour map

of the attitude of the acoustic pressure before and after control. It indicates significant noise reduction with optimal impedance factor.

The contour maps of the amplitude of the acoustic pressure u are displayed in Figure 3 for   = and optimal impedance factor   =   .
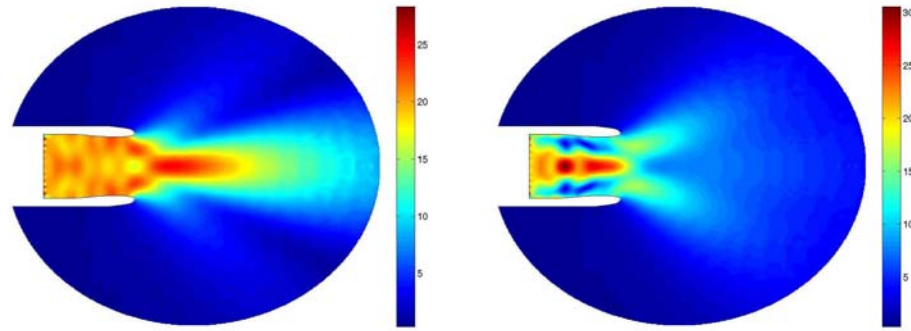

Figure 2. The contour maps of $|E(u(\ ))|$ and $|E(u(\ \ ))|$: $\eta = 1, \xi = 1$

Numerical solutions for the underlying stochastic partial differential equations (SPDES)

Because of the randomness of k, the Helmholtz equation is a stochastic partial differential equation. The Monte Carlo method is a generally applicable solution method for stochastic problems. However, it is a well-known fact that as the accuracy requirement is increased, the number of realizations to be generated and deterministic problems to be solved grows far too rapidly. In [3], We constructed a finite element method based on polynomial chaos expansion (PCE) and obtained the following rigorous error estimate.

Theorem 1. Assume that there exists a unique solution u for (1)-(2) and u^h is the finite element solution with linear finite element spaces. Then

$$\geq u - u_h \geq_{-1,p,1} \quad A(\ )_{K^{-1}}^{N,K} + B(\ )_{2^N} \geq \ \geq_{,-p+ \ ,1} + Ch \geq \ \geq_{,-p,2}$$

where h is the mesh size of the triangulations, N is the degree of polynomial, K is the dimension of Gaussian in the polynomial chaos expansion and C is a constant independent of h, N and K.

In many cases, the wavenumber is treated as a constant and a random noise forcing term is added in Equation (1) instead to count for model uncertainty. We have studied the finite element method for such a problem and obtained rigorous error estimates(see [4] for details).

Conclusion and future research

We posed the problem of optimal control and design optimization of acoustic liner under uncertainty to reduce the radiated engine noise given the source. Our numerical experiments

show that the choice of optimal impedance factor results in reducing the far-field noise level significantly. We have also constructed numerical algorithms to solve the underlying stochastic partial differential equations and obtained rigorous error estimates

In future research, we plan to study the shape design of the fan inlet to reduce the noise radiation with uncertain wave numbers. We also plan to study the stochastic problem with the wave number as a random field, not just a random variable.

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